

Synthetic Stone Duality

Freek Geerligs, jww Reid Barton, Felix Cherubini, Thierry Coquand & Hugo Moeneclaey.

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- Another axiom system (Condensed Type Theory) with the same inspiration has been studied by Barton & Commelin.
- The axioms have a model in a constructive metatheory (upcoming work by Coquand, Höfer & Sattler).

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Countably presented Boolean algebras

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Example

The initial Boolean algebra 2 is countably presented.

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$$\text{isStone}(X) := \sum_{B:\text{Boole}} X = Sp(B)$$

If $\text{isStone}(X)$ is inhabited, the type X is called Stone.

Examples of Stone spaces

Example (Cantor space)

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Axioms

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Axiom 2: Surjections are Formal surjections

A map $f : B \rightarrow C$ in Boole is injective iff the corresponding map $(-) \circ f : Sp(C) \rightarrow Sp(B)$ is surjective.

Axiom 3: Local choice

Given S Stone, E, F arbitrary, $f : S \rightarrow F$ and $e : E \twoheadrightarrow F$ surjective,

$$\begin{array}{ccc} & & E \\ & & \downarrow e \\ S & \xrightarrow{f} & F \end{array}$$

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Axiom 4: Dependent choice

Given a sequence of types $(X_n)_{n:\mathbb{N}}$ with surjections $X_n \twoheadrightarrow X_{n-1}$, all the limit projection maps $X \twoheadrightarrow X_n$ are surjective.

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For any $f : Y \rightarrow X$ and $A \subseteq X$ open, we have that $f^{-1}(A)$ is open.

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$$y \in f^{-1}(A) \leftrightarrow f(y) \in A$$

Not all closed propositions are decidable.

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For any $f : 2^{\mathbb{N}} \rightarrow 2$, the value of $f(\alpha)$ only depends on finitely many values of α .

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It is **not** the case that for any $\alpha : 2^{\mathbb{N}}$ we can decide
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Corollary

Stone spaces are sequential limits of finite sets.

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Corollary (Of `isStone` being a proposition)

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For S_n a tower of finite sets, we have $\|\lim_{n:\mathbb{N}} S_n\| \leftrightarrow \forall_{n:\mathbb{N}} \|S_n\|$.

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Corollary (Classification of closed subsets of Stone spaces)

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Corollary (transitivity of closedness)

Closed is closed under Σ .

Summary of topological results

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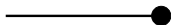
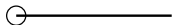
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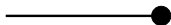
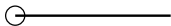
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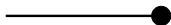


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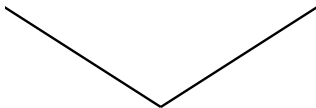
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Lemma (Localization sends topological paths to HoTT paths)

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The topologist's sine curve

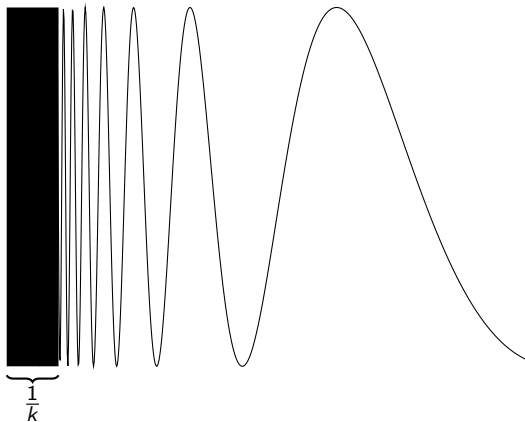
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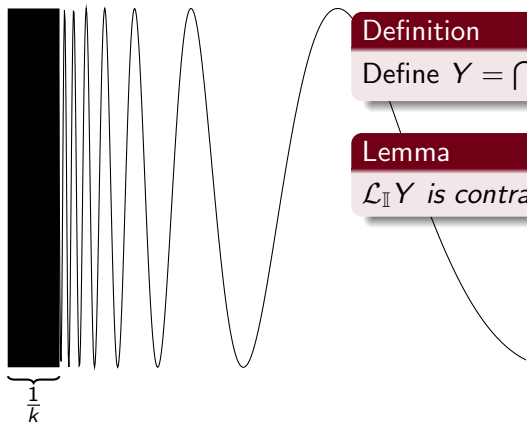
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Links

- All (WIP) papers: github.com/felixwellen/synthetic-zariski.
- WIP formalization: github.com/Freek98/FormalizationSSD
- AI formalization setup: github.com/Freek98/AutoFormalizationSetup